

Sequences & Series Review

Name: Key

skip first  $\rightarrow \sum_{m=7}^{18} 4m - 3$   
 6 terms  

$$S_n = \frac{18-6}{2} (a_7 + a_{18})$$

$$= \frac{12}{2} (25 + 69) = 564$$

$$\sum_{n=1}^{10} \frac{4^n}{3} = \frac{1}{3} (4)^n$$

$$S_n = a_1 \left( \frac{1-r^n}{1-r} \right)$$

$$= \frac{4}{3} \left( \frac{1-(4)^{10}}{1-4} \right) = 466033 \frac{1}{3}$$

If  $a_n = \frac{1}{2}n + 6$ , find  $S_7$ .

$$\sum_{n=1}^7 \left( \frac{1}{2}n + 6 \right) = \frac{7}{2} (6.5 + 9.5)$$

$$= 56$$

If  $a_n = \frac{3^n}{2}$ , find  $S_9$ .

$$= \frac{1}{2} (3)^n$$

$$\sum_{n=1}^9 \frac{1}{2} (3)^n = \frac{3}{2} \left( \frac{1-(3)^9}{1-3} \right)$$

$$= 14761.5$$

$a_n = \frac{7}{6} a_{n-1}$   
 \* recursive  
 Convergent or Divergent? Explain.  
 $r = \frac{7}{6} > 1$ , so series diverges  
 (each term is getting bigger)

$a_n = \frac{a_{n-1} + 7}{2}$   
 $= \frac{1}{2} (a_{n-1}) + \frac{7}{2}$   
 Convergent or Divergent? Explain.  
 Each term is going to be at least  $\frac{7}{2} = 3.5$ , so diverges

If  $a_{13} = 11$  and  $a_n = a_{n-1} + 5$ , find  $a_{17}$ .

$\downarrow$   
 recursive  $\Rightarrow +5$  each term

$$a_{14} = a_{13} + 5 = 11 + 5 = 16$$

$$a_{15} = 21 \quad / \quad a_{16} = 26 \quad / \quad a_{17} = 31$$

If  $a_3 = 5$  and  $a_n = -3(a_{n-1})$ , find  $a_9$ .

$\downarrow$   
 recursive  $\Rightarrow \times -3$  each time

$$a_4 = a_3 \cdot (-3) = -15$$

$$a_5 = 45 \dots a_9 = 3645$$

Find the common difference, the 11<sup>th</sup> term, the explicit formula, and the recursive formula.

1.  $13, 27, 41, 55, \dots$

$$a_n = 13 + 14(n-1)$$

$$a_{11} = 153$$

2.  $-38, -48, -58, -68, \dots$

$$a_n = -38 - 10(n-1)$$

$$a_{11} = -138$$

Given two terms in an arithmetic sequence find the common difference, the explicit formula, and the recursive formula.

3.  $a_{16} = -68, a_{35} = -182$

$$\frac{(-182) - (-68)}{(35) - (16)} = -6$$

opposite b/c I'm working backwards

$$a_1 = a_{16} + 6(15)$$

$$= -68 + 6(15) = 22$$

$$a_n = 22 - 6(n-1)$$

4.  $a_{10} = 54, a_{33} = 123$

$$\frac{(123) - (54)}{(33) - (10)} = 3$$

opposite

$$a_1 = a_{10} - 3(9) = 54 - 3(9) = 27$$

$$a_n = 27 + 3(n-1)$$

Find the common ratio, the 8<sup>th</sup> term, the explicit formula, and the recursive formula.

5.  $2, -12, 72, -432, \dots$

$$a_n = 2(-6)^{n-1}$$

$$a_8 = 2(-6)^7 = -559872$$

6.  $4, 16, 64, 256, \dots$

$$a_n = 4(4)^{n-1}$$

$$a_8 = 4(4)^7 = 65536$$

Given two terms in a geometric sequence find the common ratio, the explicit formula, and the recursive formula.

7.  $a_2 = 24, a_5 = -5184$

$$a_5 = a_2(r)^3$$

$$-5184 = 24(r)^3$$

$$-216 = r^3$$

$$-6 = r$$

Work backwards to find  $a_1 = -4$

$$a_n = -4(-6)^{n-1}$$

8.  $a_1 = -2, a_5 = -2592$

$$a_5 = a_1(r)^4$$

$$-2592 = -2(r)^4$$

$$1296 = r^4$$

$$6 = r$$

$$a_n = -2(6)^{n-1}$$

**Rewrite the sequence using sigma notation.**

1.  $-3, 7, 17, 27, 37, 47, 57, 67, 77, 87, 97, 107, 117$

2.  $3, 15, 75, 375, 1875, 9375$

**Find the sum of the first  $n$  terms of the sequence. The sequence is either arithmetic or geometric.**

3.  $3, 12, 48, 192, \dots$   $n = 14$

4.  $17, 21, 25, 29, \dots$   $n = 16$

5.  $-1, -5, -25, -125, \dots, -15625$

6.  $26, 34, 42, 50, \dots, 394$

7.  $32, 42, 52, 62, \dots, 112$

8.  $-3, 18, -108, 648, \dots, -139968$

9. An auditorium has 30 rows with 10 seats in the first row, 12 in the second row, 14 in the third row, and so forth. How many seats

10, 12, 14, ...

$$a_n = 10 + 2(n-1) \\ = 8 + 2n$$

$$\sum_{n=1}^{30} (8 + 2n) = \frac{30}{2} (a_1 + a_{30}) \\ = 15(10 + 68) = 1170$$

10. A brick staircase has a total of 20 steps. The bottom step requires 101 bricks. Each successive step requires 5 less bricks than the prior one. How many bricks are required to build the staircase?

101, 96, 91, ...

$$a_n = 101 - 5(n-1) \\ = 106 - 5n$$

$$\sum_{n=1}^{20} (106 - 5n) = \frac{20}{2} (a_1 + a_{20}) \\ = 10(101 + 6) = 1070$$

11. A business makes a \$15,000 profit during its first year. If the yearly profit increases by \$10,000 in each subsequent year, what will the profit be in the tenth year? What will be the total profit for the first ten year?

$$a_n = 15000 + 10000(n-1) \\ = 5000 + 10000n$$

$$\sum_{n=1}^{10} (5000 + 10000n) \\ = \frac{10}{2} (15000 + 105000) \\ = 600,000$$

$$a_{10} = 105000 \text{ (profit for } 10^{\text{th}} \text{ year alone)}$$

12. A ball is dropped from a height of 10 feet. On each bounce it rises to half its previous height. When the ball hits the ground for the seventh time, how far has it traveled?

10, 5, 2.5, ...

$$a_n = 10\left(\frac{1}{2}\right)^{n-1}$$

$$\sum_{n=1}^7 10\left(\frac{1}{2}\right)^{n-1} = 10 \left( \frac{1 - \left(\frac{1}{2}\right)^7}{1 - \left(\frac{1}{2}\right)} \right) \\ = 19.84375 \text{ ft}$$

13. If you are paid a salary of 1 cent on the first day, 2 cents on the second day, and your salary continues to double each day, how much will you earn in 20 days?

.01, .02, .04, ...

$$a_n = .01(2)^{n-1}$$

$$\sum_{n=1}^{20} .01(2)^{n-1} = .01 \left( \frac{1 - (2)^{20}}{1 - 2} \right) \\ = \$10485.75$$