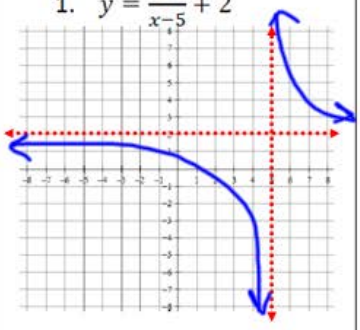
10.1 Graph Rational Functions

PRACTICE

Graph the following and write the equations of the horizontal asymptote and vertical asymptote.

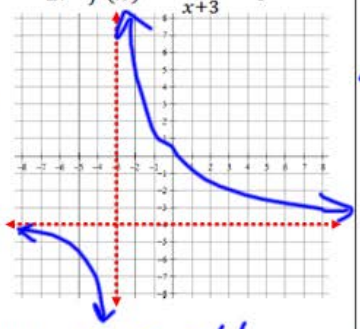
1. $y = \frac{1}{x-5} + 2$



HA: $y = 2$

VA: $x = 5$

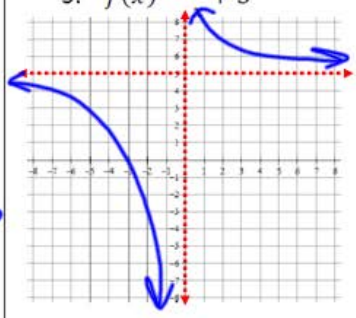
2. $f(x) = \frac{1}{x+3} - 4$



HA: $y = -4$

VA: $x = -3$

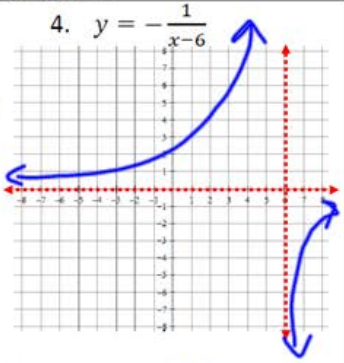
3. $f(x) = \frac{1}{x} + 5$



HA: $y = 5$

VA: $x = 0$

4. $y = -\frac{1}{x-6}$

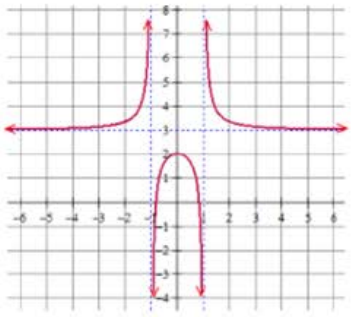


HA: $y = 0$

VA: $x = 6$

Given the graph of a rational function, find the asymptotes and intercepts if they exist.

5.



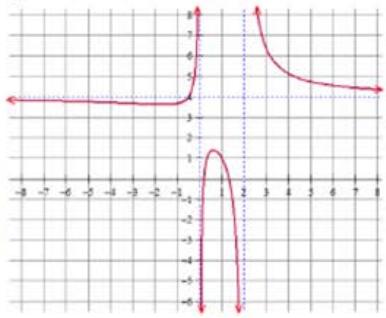
VA: $x = -1$, $x = 1$

HA: $y = 3$

x-int: $\approx (0.9, 0)$, $\approx (-0.9, 0)$

y-int: $(0, 2)$

6.



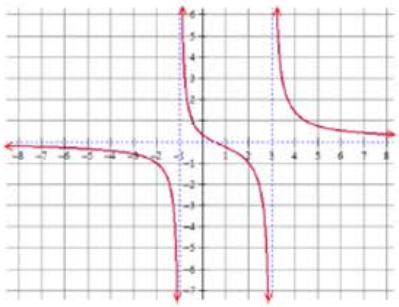
VA: $x = 0$, $x = 2$

HA: $y = 4$

x-int: $\approx (0.2, 0)$, $\approx (1.4, 0)$

y-int: none

7.



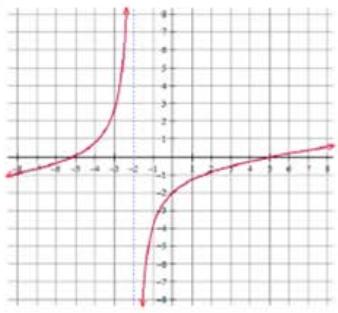
VA: $x = -1$, $x = 3$

HA: $y = 0$

x-int: $(0.5, 0)$

y-int: $\approx (0, 0.3)$

8.



VA: $x = -2$

HA: none

x-int: $(-5, 0)$, $(5, 0)$

y-int: $(0, -2)$

Find the horizontal and vertical asymptotes if they exist.

9. $f(x) = \frac{4x^2 + 7x - 18}{x^2 - 25}$

VA: $(x+5)(x-5) = 0$, $x = -5, 5$

HA: $y = \frac{4}{1} = 4$

10. $f(x) = \frac{3x^2 + 2x - 5}{3x^3 - 27x}$

$3x(x^2 - 9) = 0$

VA: $3x(x+3)(x-3) = 0$, $x = 0, -3, 3$

HA: $y = 0$

11. $f(x) = \frac{x^4}{x^2 - 3x - 40}$

$(x-8)(x+5) = 0$

VA: $x = 8, -5$

HA: none

Find the x-intercept(s) and y-intercept if they exist.

12. $f(x) = \dfrac{4x-7}{2x^2+1}$

$4x-7=0$, $4x=7$, $x=\frac{7}{4}$

x-intercept(s): $(\frac{7}{4}, 0)$

y-intercept: $\dfrac{4(0)-7}{2(0)^2+1} = -7$

$(0, -7)$

13. $f(x) = \dfrac{x^2-20}{5x^2-4x-9}$

$x^2-20=0$, $x^2=20$, $x=\pm\sqrt{20}$, $x=\pm 2\sqrt{5}$

x-intercept(s): $(-2\sqrt{5}, 0)\ (2\sqrt{5}, 0)$

y-intercept: $\dfrac{0^2-20}{5(0)^2-4(0)-9} = \dfrac{-20}{-9}$

$(0, \frac{20}{9})$

14. $f(x) = \dfrac{x^2+6x-16}{5x^4-3x-8}$

$(x+8)(x-2)=0$, $x=-8, 2$

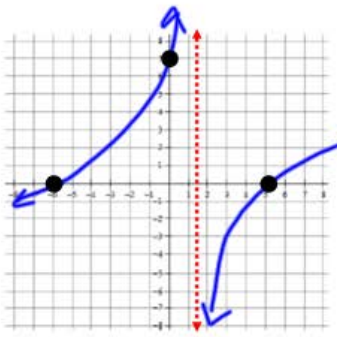
x-intercept(s): $(-8, 0)\ (2, 0)$

y-intercept: $\dfrac{0^2+6(0)-16}{5(0)^4-3(0)-8} = \dfrac{-16}{-8} = 2$

$(0, 2)$

Find all asymptotes and intercepts. Mark them on the graph. Use the graphing calculator to sketch the function.

15. $f(x) = \dfrac{x^2+x-30}{3x-5}$



VA: $3x-5=0$, $3x=5$, $x=\frac{5}{3}$

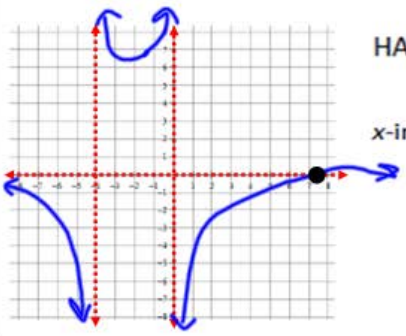
HA: none

x-int: $(x+6)(x-5)=0$, $x=-6, 5$, $(-6, 0)\ (5, 0)$

y-int: $\dfrac{0^2+0-30}{3(0)-5} = \dfrac{-30}{-5} = 6$

$(0, 6)$

16. $f(x) = \dfrac{2x-15}{x^2+4x}$



VA: $x(x+4)=0$, $x=0, -4$

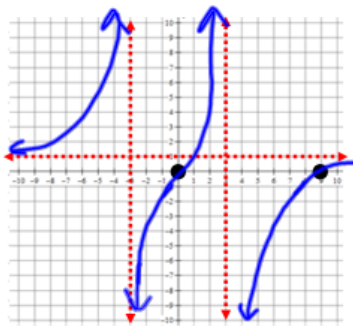
HA: $y=0$

x-int: $2x-15=0$, $2x=15$, $x=\frac{15}{2}$, $(\frac{15}{2}, 0)$

y-int: $\dfrac{2(0)-15}{0^2+4(0)} = \dfrac{-15}{0}$

none!

17. $f(x) = \dfrac{x^2-9x}{x^2-9}$



VA: $(x+3)(x-3)=0$, $x=3, -3$

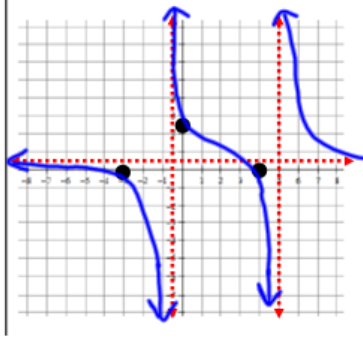
HA: $y=1$

x-int: $x(x-9)=0$, $x=0, 9$, $(0,0)\ (9,0)$

y-int: $\dfrac{0^2-9(0)}{0^2-9} = \dfrac{0}{-9} = 0$

$(0, 0)$

18. $f(x) = \dfrac{x^2-x-12}{2x^2-9x-5}$



VA: $(2x+1)(x-5)=0$, $x=-\frac{1}{2}, 5$

HA: $y=\frac{1}{2}$

x-int: $(x-4)(x+3)=0$, $x=4, -3$, $(4,0)\ (-3,0)$

y-int: $\dfrac{0^2-0-12}{2(0)^2-9(0)-5} = \dfrac{-12}{-5} = \dfrac{12}{5}$

$(0, \frac{12}{5})$